Problem 9.5

A 65.0 kg boy and a 40.0 kg girl, both wearing roller blades (this implies you can ignore friction), stand stationary facing one another. The girl pushes the boy sending him at 2.90 m/s to the west (defined as the negative x-direction).

a.) Describe the resulting motion.

Due to N.T.L., the force the girl exerts on the boy is equal and opposite the force the boy exerts on the girl. Being a kind of collision, the time interval is the same for both exertions of force, which means the impulses ($\vec{F}\Delta t$ each) are equal and opposite. From that, we can determine the girl's recoil velocity.

From the boy's impulse equation, keeping track of the vector nature:

$$\begin{aligned}
\vec{F}\Delta t &= \Delta\vec{p}_{boy} \\
&= \vec{p}_{2,boy} - \vec{p}_{1,boy} \\
&= m\left|v_{2,boy}\right|\left(-\hat{i}\right) - 0 \\
&= (65.0 \text{ kg})(2.90 \text{ m/s})\left(-\hat{i}\right) \\
&= (-189 \text{ kg}\bullet\text{m/s})\hat{i}
\end{aligned}$$

Minor point: There is a bit of a subtlety here that should be pointed out. Momentum is a *vector.* If the situation is one-dimensional, though, you don't really need to include the unit vector but you do need to keep track of the signs. That means it would have been acceptable to write that first set of equation as:

$$
\begin{aligned}
F_x \Delta t &= \Delta p_{boy,x} \\
&= p_{2,boy,x} - p_{1,boy,x} \\
&= m\left(-v_{2,boy,x}\right) - 0 \\
&= (65.0 \text{ kg})(-2.90 \text{ m/s}) \\
&= -189 \text{ kg} \bullet \text{m/s}
\end{aligned}
$$

Knowing that the girls' impulse is equal to *minus* the boy's impulse, we can write:

$$
\begin{aligned}
-(-189 \text{ kg} \bullet \text{m/s})\hat{i} &= \Delta \vec{p}_{girl} \\
&= m\left|v_{2,girl}\right|\hat{i} - 0 \\
&= (40.0 \text{ kg})\left|v_{2,girl}\right|\hat{i} \\
\Rightarrow \quad v_{2,girl} &= (4.73 \text{ m/s})\hat{i}
\end{aligned}
$$

b.) How much potential energy in the girl's body is provided to the system?

The girl provides all the energy that will become kinetic energy in the system. That is:

$$\begin{aligned} U_{girl} &= \frac{1}{2}m_{boy}\left(v_{boy}\right)^2 + \frac{1}{2}m_{girl}\left(v_{girl}\right)^2 \\ &= \frac{1}{2}(65.0 \text{ kg})(-2.90 \text{ m/s})^2 + \frac{1}{2}(40.0 \text{ kg})(4.71 \text{ m/s})^2 \\ &= 717 \text{ J} \end{aligned}$$

c.) Is the *momentum* conserved in the pushing process?

Momentum is conserved (i.e., doesn't change in time) in a particular direction if there are *only internal forces* (which is to say, forces that are generated as the consequence of the interaction of the pieces in the system) acting in that direction. In this system, the only forces acting along the line of motion are the girl pushing the boy and reaction force of the boy pushing back on the girl. These are *internal forces*, so momentum is *conserved*. Additionally, as nothing was moving to start with, the momentum along the line of motion must always sum to ZERO as that was the initial momentum of everything in that direction at the start.

d.) How can the system be conserved in momentum if the forces acting are large?

What is important when deciding if a system's momentum is conserved is not the magnitude of the forces that are acting in the system (along the line of interest) but whether the forces are internal to the system (which is to say, the consequence of the interaction of the pieces of the system). If that be the case, the impulse provided by one force will be equal and opposite the impulse provided by the other, and the net impulse will be zero. As a *net impulse* is required to *change the momentum* of a system in a particular direction, and as no net impulse is present here, momentum is conserved no matter how large the internal forces are.

e.) How can the system be conserved in momentum if there is no motion to start with but all sorts of motion afterwards.

Momentum is a vector. As such, it is quite possible to have two momentum vectors that are quite large whose *magnitudes* are the same but whose directions are opposite (the boy's final momentum is $-(189 \text{ kg} \bullet \text{m/s})\hat{\text{i}}$, the girl's final momentum is $+(189 \text{ kg} \bullet \text{m/s})\hat{\text{i}}$).

In such cases, the sum of the two final momenta is zero, which was the momentum in the system at the beginning, and momentum is conserved!